Interpretation of the Tresca yield criterion

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Consider a material element that is loaded to a stress state

$$\sigma = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

and let us assume that this (uniaxial) stress state is almost enough to cause the material to plastically deform, or yield (perhaps the yield stress $\sigma_{\text{yld}} = 54\,\text{MPa}$, for example). If the loading is changed so that the new stress state σ' is

$$\sigma' = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

will the material yield under the Tresca (maximum shear) criterion? How about if the material is loaded to the stress state

$$\sigma'' = \begin{bmatrix} 50 & 0 & 0 \\ 0 & -20 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

Will the material yield in this case? There are several ways to solve this problem.

1. The first approach is to simply apply the equation associated with the Tresca criterion: is $\sigma_1 - \sigma_3 > \sigma_{\text{yld}}$? If so, the material will yield. Note that σ_1, σ_2, and σ_3 here are the *principal stresses*, the normal stresses on an element that is oriented so that any shear stresses vanish. Also, these principal stresses are always labeled from largest to smallest, so that $\sigma_1 > \sigma_2 > \sigma_3$.

   There are no shear stresses in any of the original loading scenarios, so the element is already in a principal stress state. The principal stresses for the σ' case are labeled as

$$\sigma_1 = \sigma_{xx} = 50\,\text{MPa}$$
$$\sigma_2 = \sigma_{yy} = 20\,\text{MPa}$$
$$\sigma_3 = \sigma_{zz} = 0\,\text{MPa}$$

   The Tresca equation is therefore

$$\sigma_1 - \sigma_3 = 50\,\text{MPa} < 54\,\text{MPa} = \sigma_{\text{yld}}$$

   so the material does not yield in this case.

   For the σ'' case, we again label the principal stresses from largest to smallest:

$$\sigma_1 = \sigma_{xx} = 50\,\text{MPa}$$

$$\sigma_2 = \sigma_{zz} = 0\,\text{MPa}$$

$$\sigma_3 = \sigma_{yy} = -20\,\text{MPa}$$

The Tresca equation is now

$$\sigma_1 - \sigma_3 = 70\,\text{MPa} > 54\,\text{MPa} = \sigma_{\text{yld}}$$

so the material is now predicted to yield.

We might have intuitively guessed that adding the 20 MPa stress orthogonal to the original 50 MPa stress increases the likelihood that the material will yield. We might also have guessed that the −20 MPa addition somehow lessens the stress on the element and makes yield less likely. *The calculations above show that neither hunch is correct.* The 20 MPa addition does not increase the chance of yielding, while the −20 MPa addition ensures that yield will occur.

2. A second way to solve the problem is to plot the stress states and compare them to the Tresca boundary for plane strain ($\sigma_{zz} = 0$). The original stress state is plotted at point A, slightly short of the yield boundary that crosses the x-axis at 54 MPa. The revised stress states σ' and σ'' are plotted at points B and C, respectively. The conclusions we draw are the same as those above, as point B lies inside the yield boundary and point C lies outside the yield boundary.

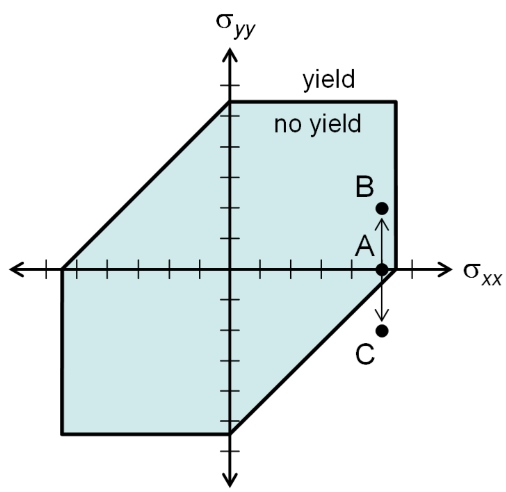

Figure 1: The Tresca boundary for plane strain, with the original (σ) and revised stress states (σ', σ'') plotted at points A, B, and C, respectively. Each division on the axes represents 10 MPa. Point C lies outside the boundary, indicating that the material will yield under the stress state σ''.

3. The third approach is more detailed, but perhaps the most satisfying. It explains the Tresca equation and the shape of the Tresca boundary. It considers the key question for the yielding of ductile materials: what is the maximum shear? We know that changing an element's orientation changes the normal and shear stresses. Since ductile materials fail in shear, it is natural to seek the element orientation that produces the maximum shear.

The original stress state (uniaxial with $\sigma_{xx} = 50\,\text{MPa}$) is shown in Figure 2 along with a couple of Mohr's circles to guide us. It should be clear that the maximum shear stress will be obtained if we reorient the element by a 45° rotation in either the x-y or x-z plane. From inspection of the Mohr's circles, the maximum shear stress $\tau_{\max} = \frac{\sigma_{xx}}{2}$. If the uniaxial stress were increased to 54 MPa, we know the material will yield; therefore, the maximum allowable shear stress is $\frac{54\,\text{MPa}}{2} = 27\,\text{MPa}$. *This is the key value by the Tresca (maximum shear) criterion; if the shear stress exceeds 27 MPa in any element orientation, the material will yield.*

The revised shear stress σ' is shown in Figure 3. The additional loading causes an additional stress of 20 MPa in the y-direction, and a reduced maximum shear stress in the x-y plane. The maximum shear stress achievable in the x-z direction is still 25 MPa, however, so the yield situation does not change.

The revised shear stress σ'' is shown in Figure 4. The maximum shear stress in the x-z direction is still 25 MPa; however, the maximum shear stress in the x-y direction is now 35 MPa. Since this value is larger than the maximum allowable shear stress of 27 MPa, the material will yield.

The three approaches are equivalent. The first and second are relatively simple and easier to use in daily practice. The third should be reviewed to understand the reasoning and assumptions underlying the Tresca criterion.

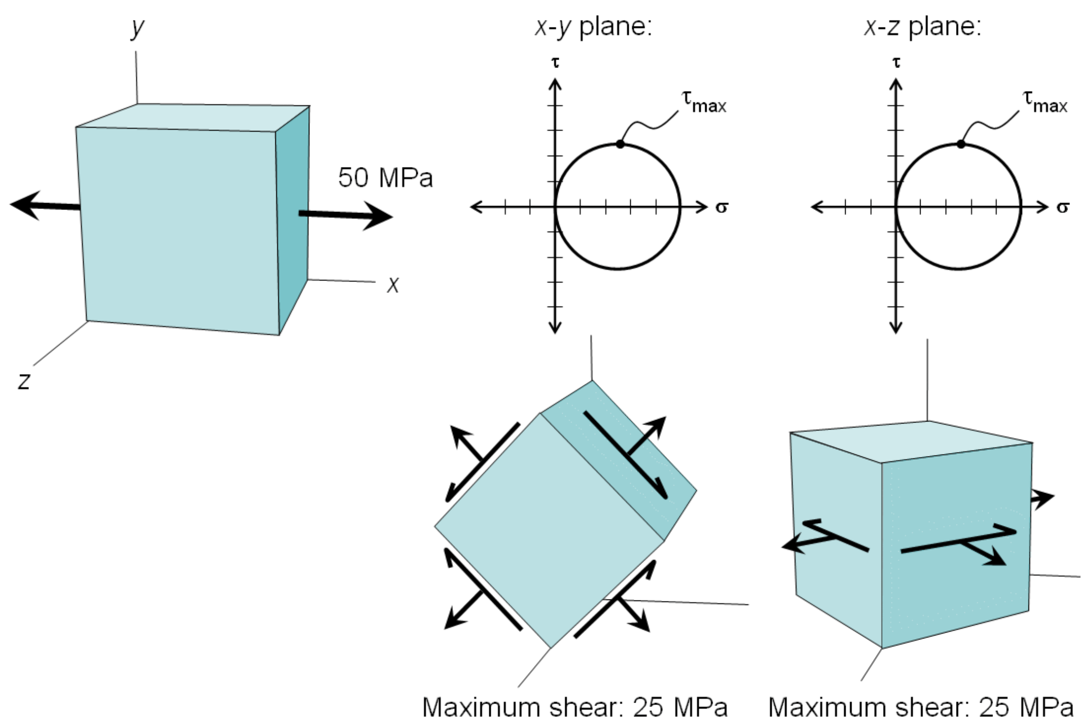

Figure 2: The original stress state σ (top left), associated Mohr's circles, and element orientations to achieve maximum shear stress (bottom). A maximum shear stress of 25 MPa can be obtained by rotating the element 45° in either the x-y or x-z plane.

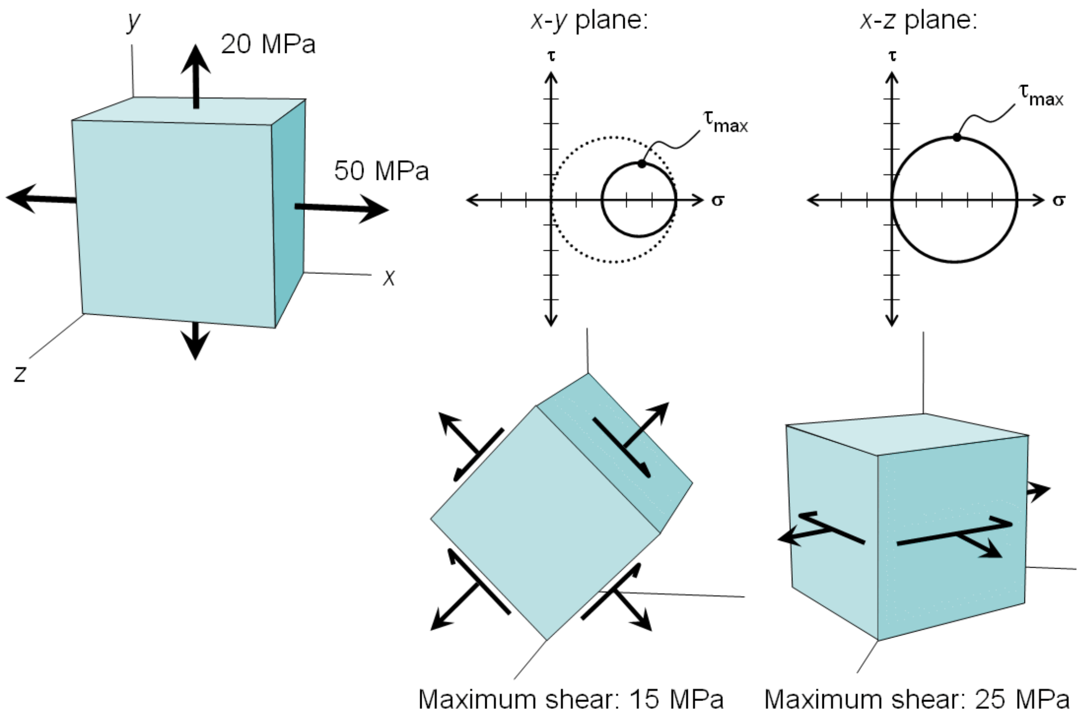

Figure 3: In the revised stress state σ', the maximum shear stress in the *x*-*y* plane is reduced to 15 MPa. However, the maximum shear stress in the *x*-*z* plane is still 25 MPa. The shear stress does not exceed 27 MPa in any orientation, so the material does not yield.

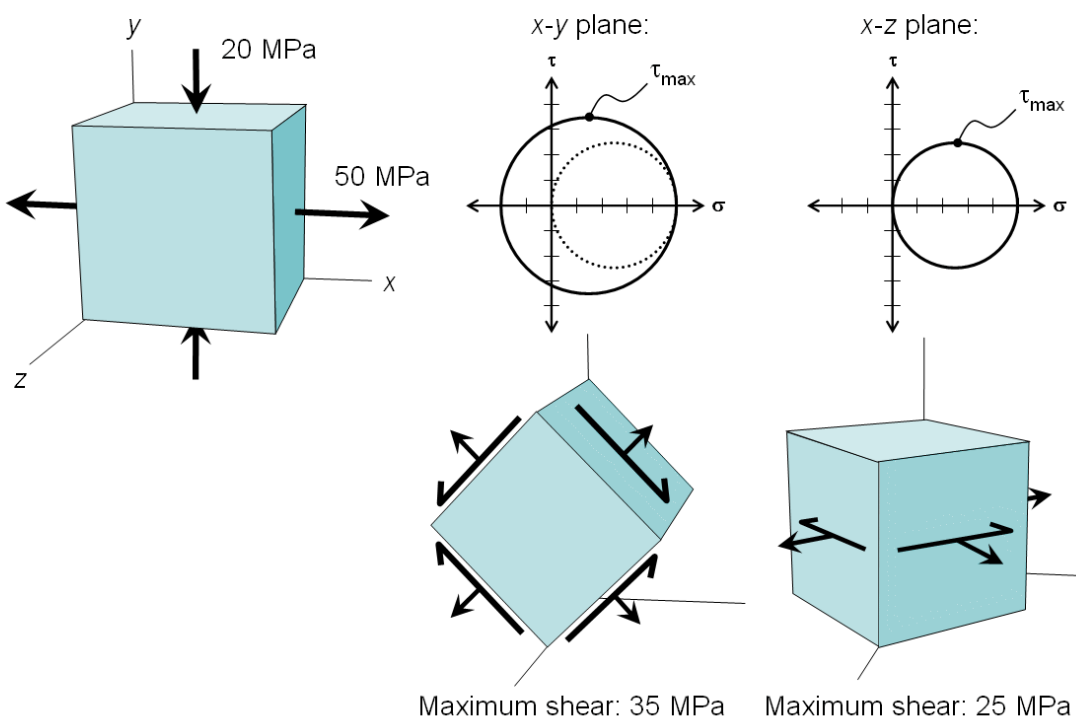

Figure 4: In the revised stress state σ'', the maximum shear stress in the *x*-*y* plane is now 35 MPa. This value is larger than the maximum allowable value of 27 MPa, so the material will yield.