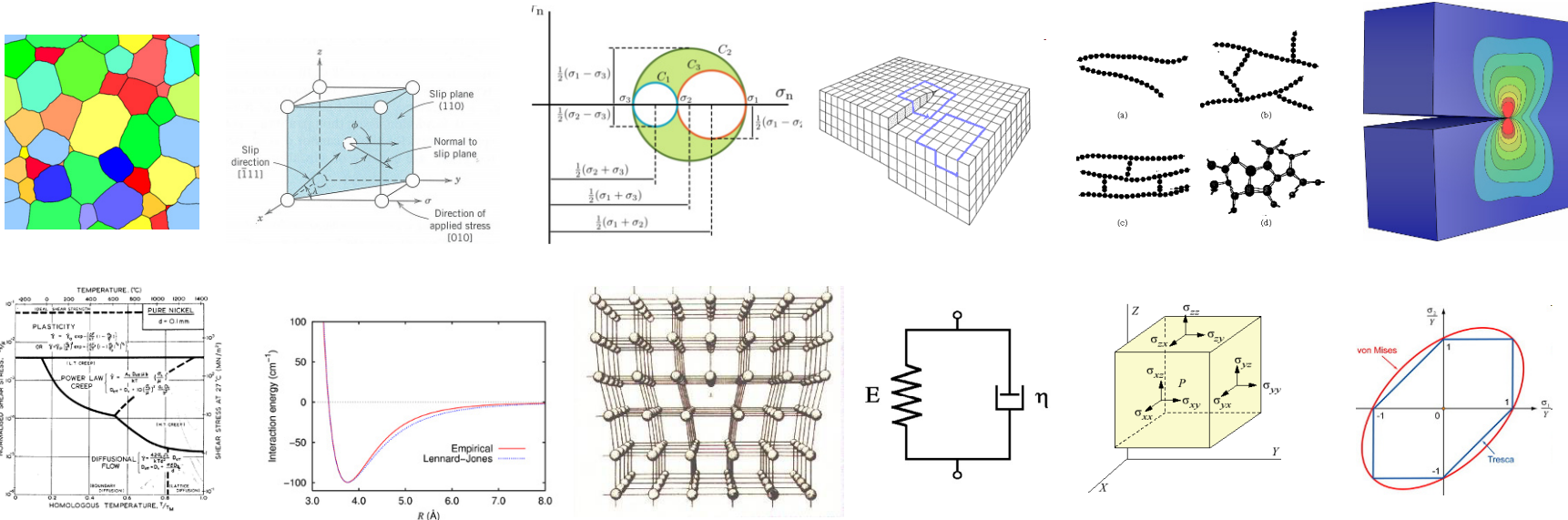


# What we should know about mechanics of materials



John Maloney

Van Vliet Group / Laboratory for Material Chemomechanics  
 Department of Materials Science and Engineering  
 Massachusetts Institute of Technology

May 3, 2013

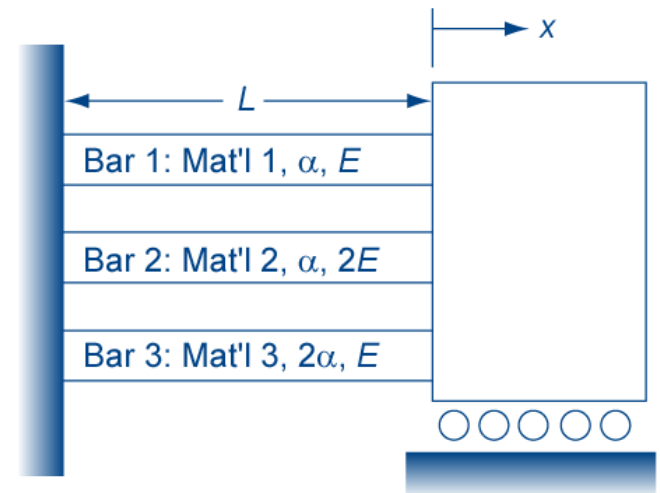
0. Every spontaneous process is driven by free energy minimization.

This generalizes our mechanical intuition: that forces are balanced when a system is in equilibrium.

Energy minimization easily explains mechanisms such as vacancy formation, splitting into partial dislocations, brittle fracture, solute strengthening, rubber elasticity, and many others.

$$dG = -S dT + F dL \quad (1-D)$$

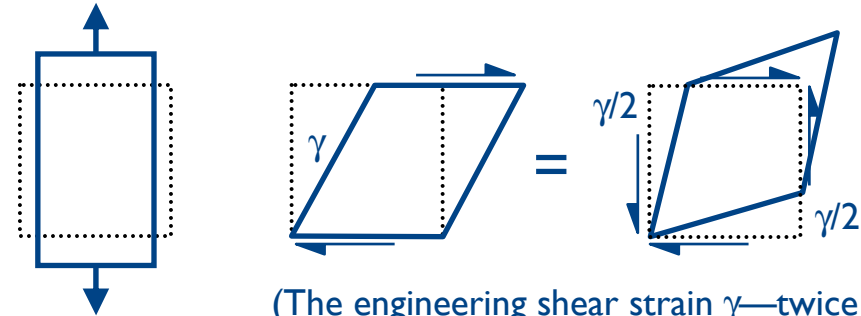
$$dG = -S dT + \sigma V d\varepsilon \quad (3-D)$$



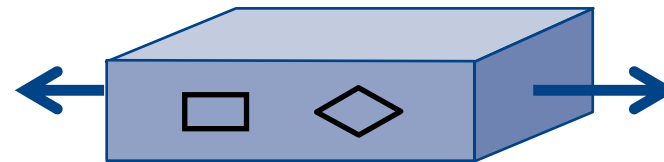
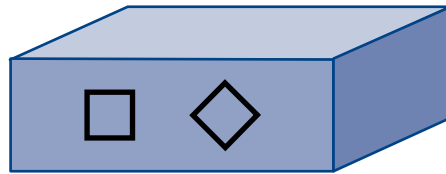
Knowing  $\Delta T$ , find  $x$ :  
a 50-minute forces problem,  
a 10-minute strain energy problem.

I. Infinitesimal elements: *normal* strain changes side *length*, while *shear* strain changes side *angle*.

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$



(The engineering shear strain  $\gamma$ —twice the tensorial strain  $\varepsilon$ —is the angular reduction in radians.)



But in finite bodies, *dilatational* (or hydrostatic) strain changes *size*, while *deviatoric* strain changes *shape*.

$$\sigma_{\text{dil}} = \frac{\sigma_{kk}}{3} = -p \quad \sigma_{\text{dev}} = \sigma - I\sigma_{\text{dil}}$$

2.  $E$  in simple Hooke's Law is restricted to uniaxial loading of long objects where lateral stresses are negligible.

$$\sigma = E\varepsilon$$

Generalized Hooke's Law works for any loading condition and geometry of an isotropic material.

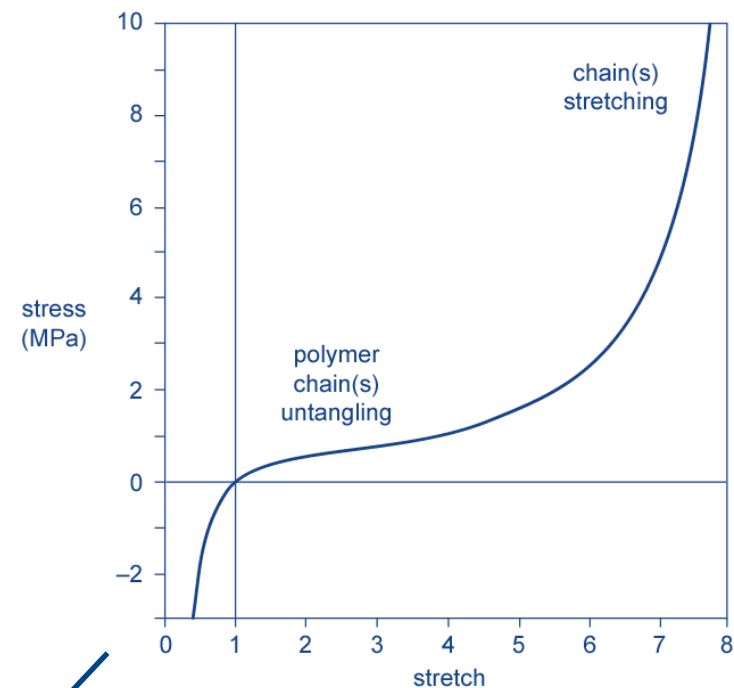
$$\varepsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} + \alpha \Delta T \delta_{ij}$$

Dilatational stress/strain is coupled by  $K$ , while deviatoric stress/strain is coupled (in the pure shear case) by  $G$ , both easily derived from generalized Hooke's Law. Both must be positive, constraining  $\nu$  to  $-1 \leq \nu \leq 0.5$ .

3. Stiffness may be dominated by *enthalpy*  $H$  (metals, ceramics, polymers below  $T_g$ ) or by *entropy*  $S$  (crosslinked polymers above  $T_g$ ).

$$E = \left( \frac{\partial^2 U}{\partial \varepsilon^2} \right) - T \left( \frac{\partial^2 S}{\partial \varepsilon^2} \right)$$

*Enthalpic* stiffness correlates with density and melting temperature, decreases slightly with increasing temperature.



*Entropic* stiffness in single molecules increases linearly with temperature. (Overall stiffness increases strongly as molecule straightens and covalent, enthalpic bond stretching dominates.)

4. The 2<sup>nd</sup>-rank strain tensor couples undeformed and deformed vectors; the 2<sup>nd</sup>-rank stress tensor couples force and surface normal vectors.

These tensors are symmetric because we discard rotation and translation.

A 4<sup>th</sup>-rank tensor couples stress and strain with a maximum of 21 independent components; this tensor is symmetric because  
(1) applying stress and recording strain is equivalent to  
(2) applying strain and recording stress.

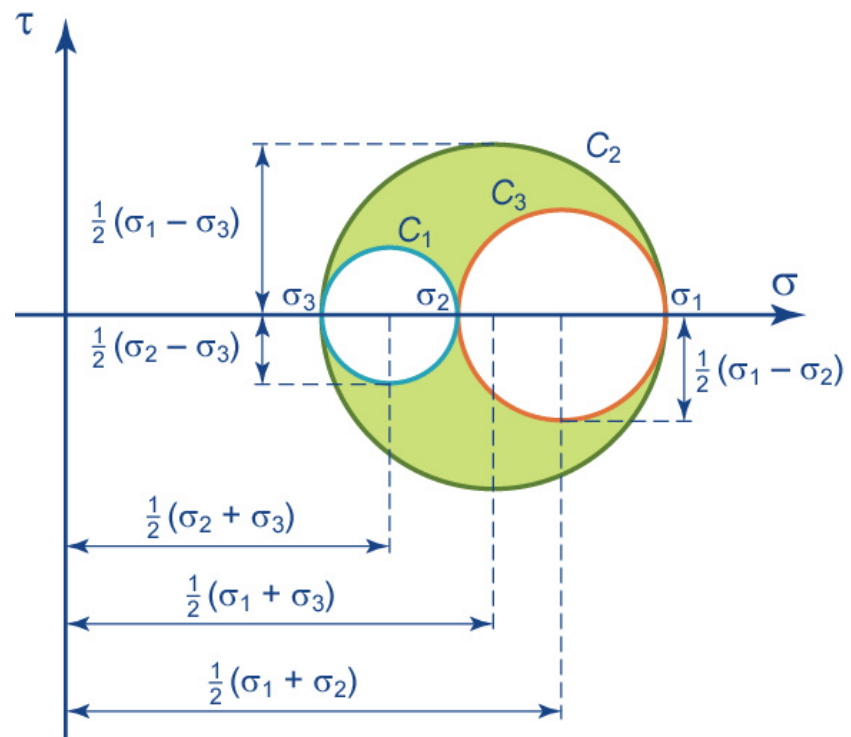
$$\sigma = C \varepsilon$$

5. Tensor transformations are essential for determining planes of maximum normal and shear stress; matrix eigenvalues correspond to principal stresses and strains.

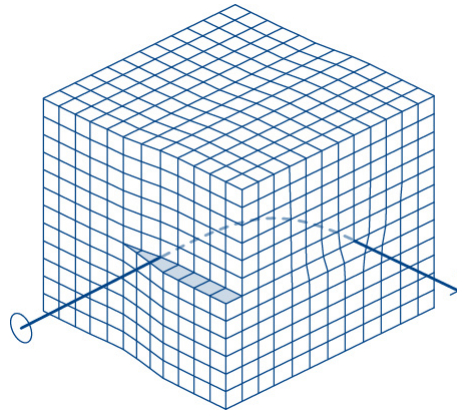
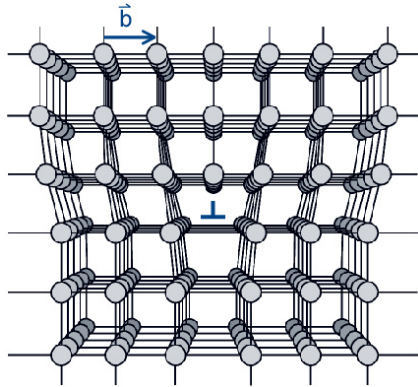
Can there be a more thrilling mathematical object than the direction cosine matrix  $a$ ? (\*\*SARCASM\*\*)

$$\sigma' = a\sigma a^T$$

Mohr's circle provides a (relatively) more enjoyable graphical interpretation.



6. Dislocations are 1-D defects that carry plasticity through a crystal (with energy  $\propto Gb^2$ ); they demarcate slipped and unslipped regions.

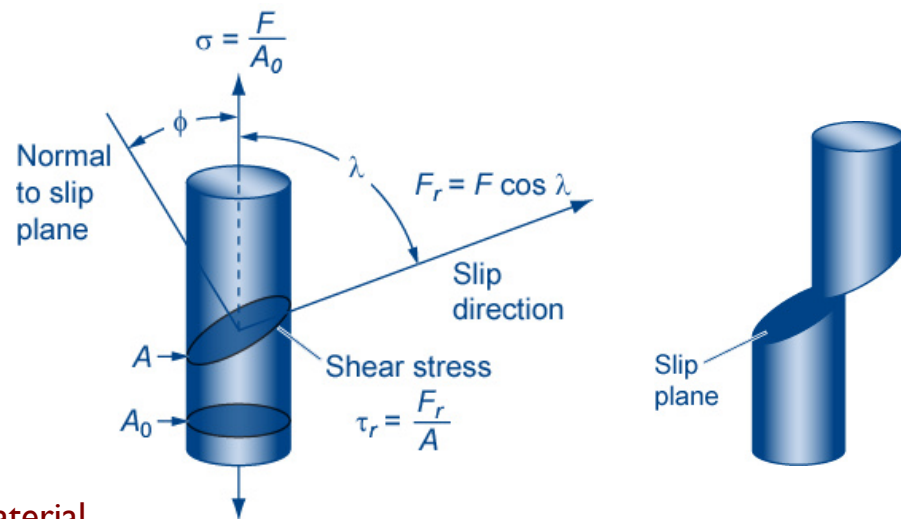


**DEFECT COUNTOFF!!!**

- 0-D: vacancies, interstitials
- 1-D: dislocations
- 2-D: grain boundaries, surfaces
- 3-D: precipitates

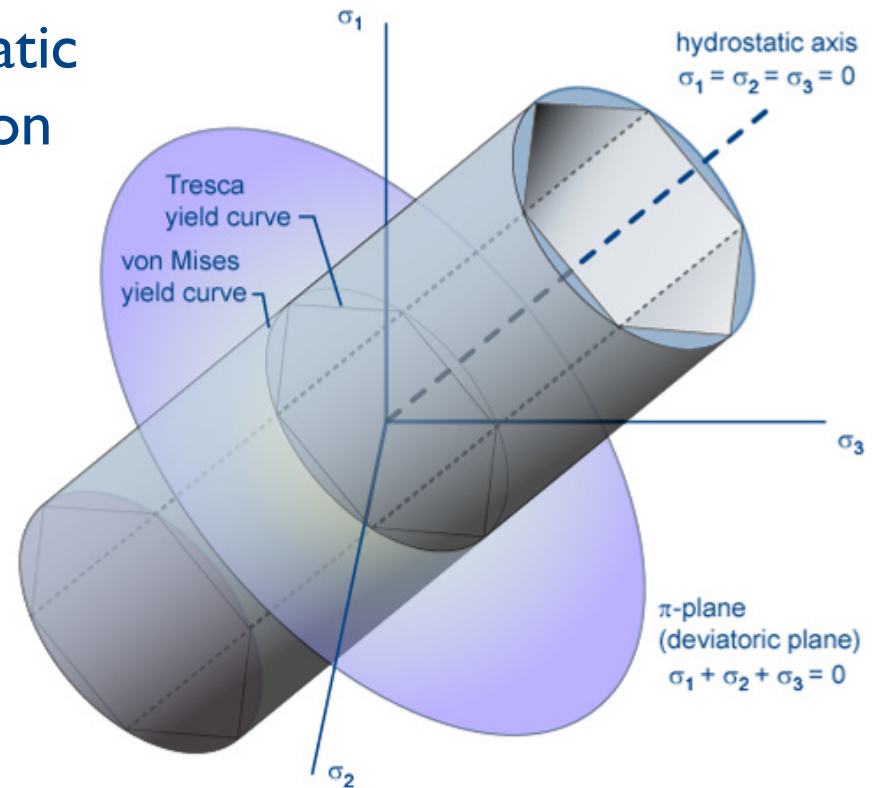
Dislocations glide at a sufficient shear stress on the most closely packed plane, in the most closely packed direction.

$$\tau_{res} = \frac{P}{A} \cos \phi \cos \lambda$$

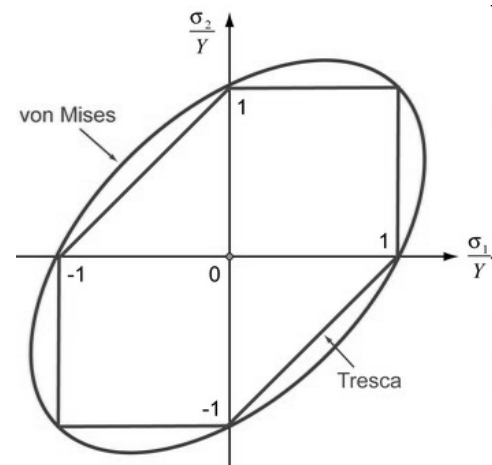




7. The minimal influence of hydrostatic (compressive dilatational) stress on plastic deformation means that a zone of safety exists around  $\sigma_x = \sigma_y = \sigma_z < 0$  for solid materials.



Our models of the zone boundary correspond to failure criteria.



## 8. To strengthen crystals, inhibit dislocation motion.

Add misfitting solute atoms that will preferentially sit in dislocation stress fields, requiring diffusion to achieve glide.

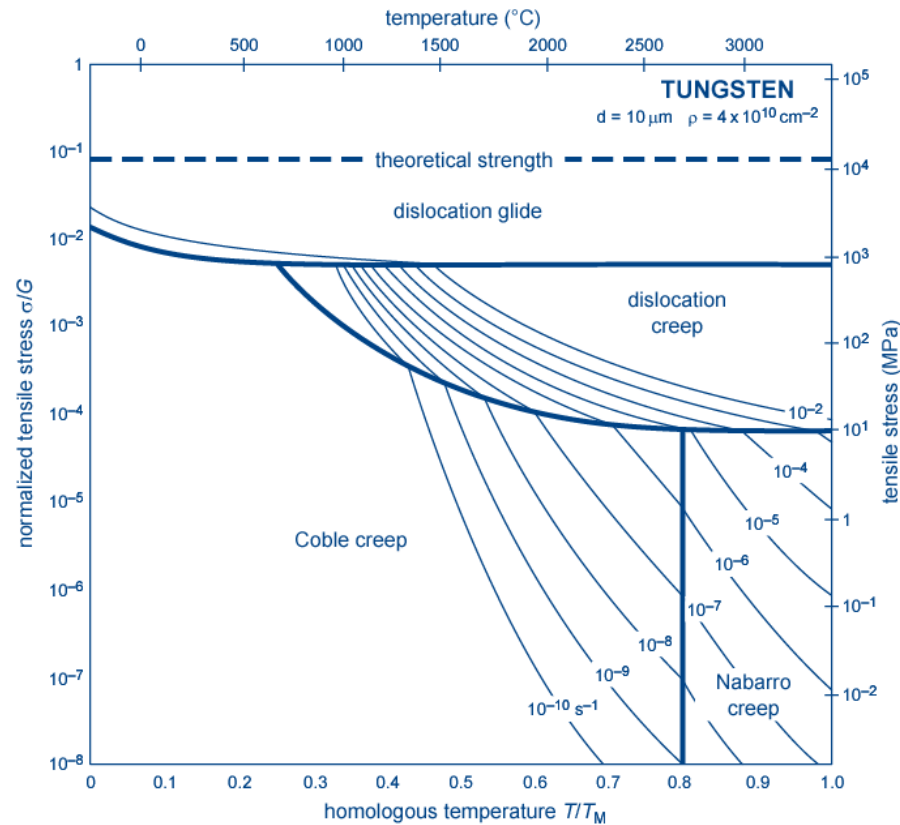
Add dislocations by cold working that will entangle each other, generate jogs, produce sessile segments, increase length.

Reduce grain size to suppress dislocation pile-up at grain boundaries.

Nucleate precipitates or add particles to force dislocations to bow around or cut through.

(Each technique has its limits.)

## 9. There is no such thing as elasticity, only negligible plasticity.

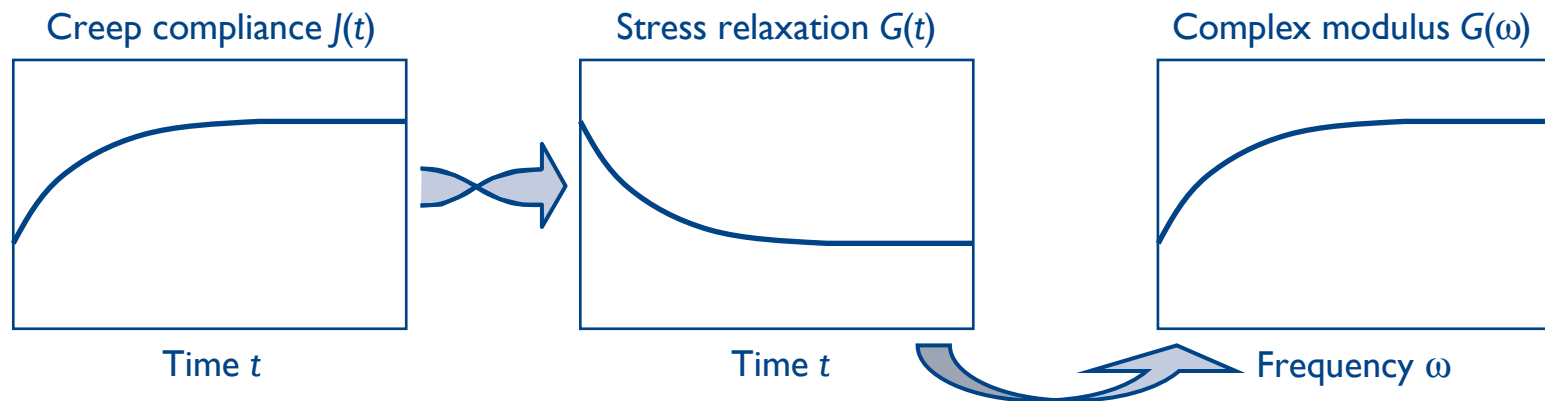
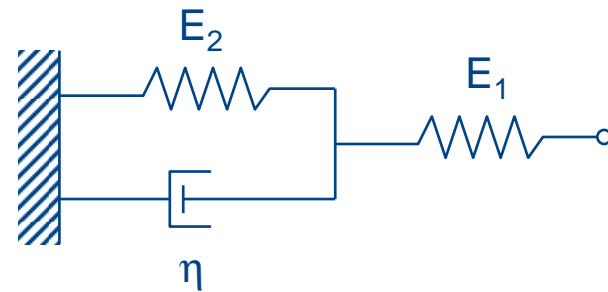


A variety of thermally activated mechanisms get in our face as the absolute temperature exceeds  $\sim T_m/2$ .

$$\dot{\epsilon} \propto \exp\left(-\frac{\Delta E}{RT}\right) \frac{\sigma^a}{d^b}$$

10. Time-dependent behavior is often modeled by arrangements of springs and dashpots, enabling a complex mechanical impedance.

Springs are purely elastic (stiffness  $E$ ); dashpots are purely viscous (stiffness  $\eta j\omega$ )



The practitioner of viscoelasticity who learns Laplace transforms is a happy practitioner.  
Stiffness of the assembly above: \_\_\_\_\_

$$\frac{1}{\frac{1}{E_1} + \frac{1}{E_2 + \eta j\omega}}$$

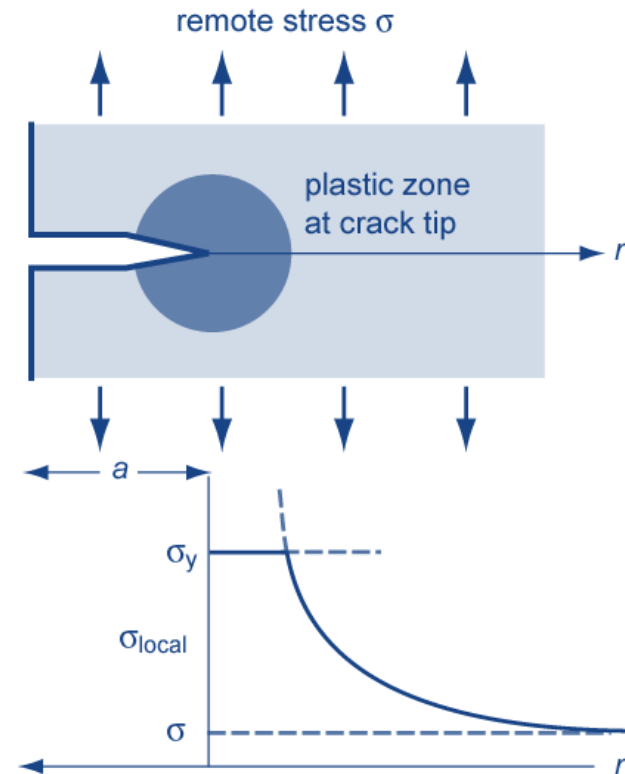
I I. Brittle fracture occurs when the stress concentration at a crack tip cannot be sufficiently blunted by plastic deformation.

When the benefit of relieving strain energy more than offsets the penalty of making more new surface, look out.

$$\sigma_{\text{crit}} = \sqrt{\frac{2E\gamma}{\pi a}}$$

Fracture toughness  $K_{IC}$  is a material property.

$$K_{IC} \propto \sigma \sqrt{\pi a_C}$$

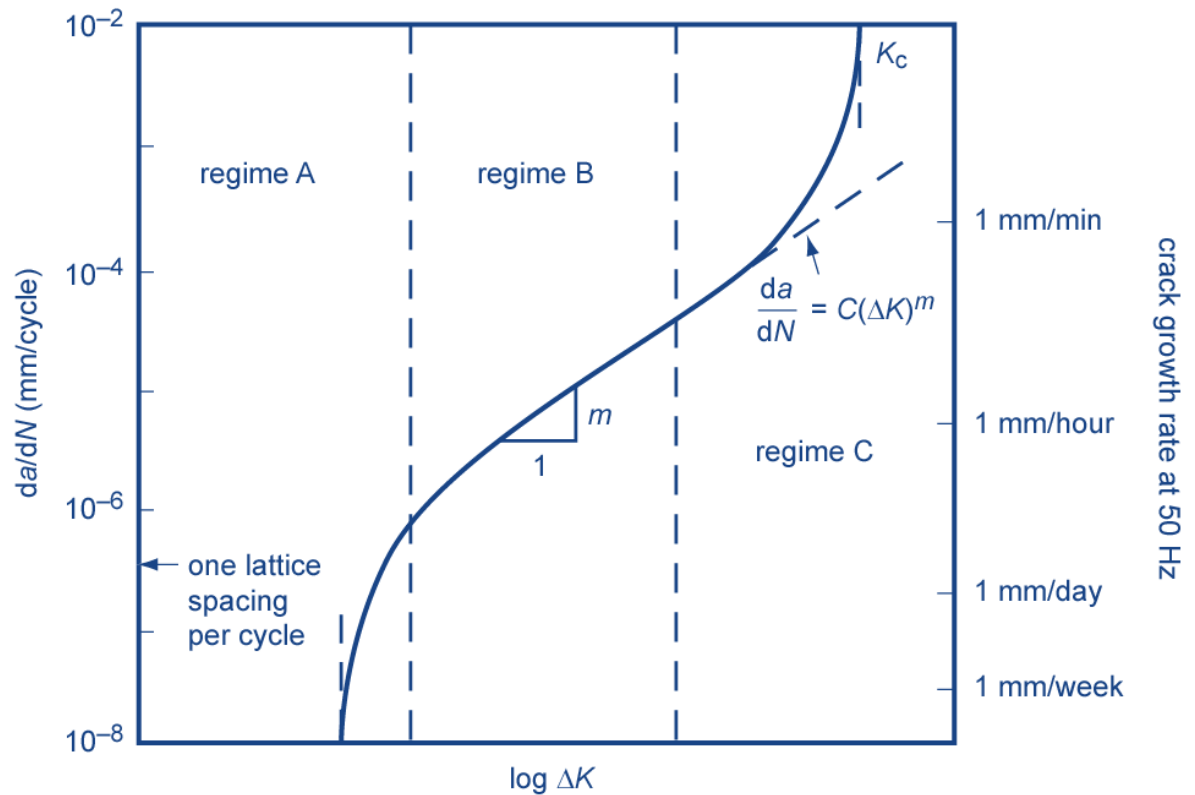


## 12. Repeated loading can resharpen and reblunt crack tips, enabling incremental crack propagation.

In the Paris law regime, crack growth rate depends predictably on the stress intensity factor  $\Delta K$ .

$$\Delta K \propto \Delta\sigma \sqrt{\pi a}$$

$$\frac{da}{dN} \propto \Delta K^m$$



13. A grab bag of topics fills out our knowledge.

Composites

Pressure vessels

Cellular solids

Elastic instability (buckling)

Further plasticity mechanisms such as twinning and GB sliding

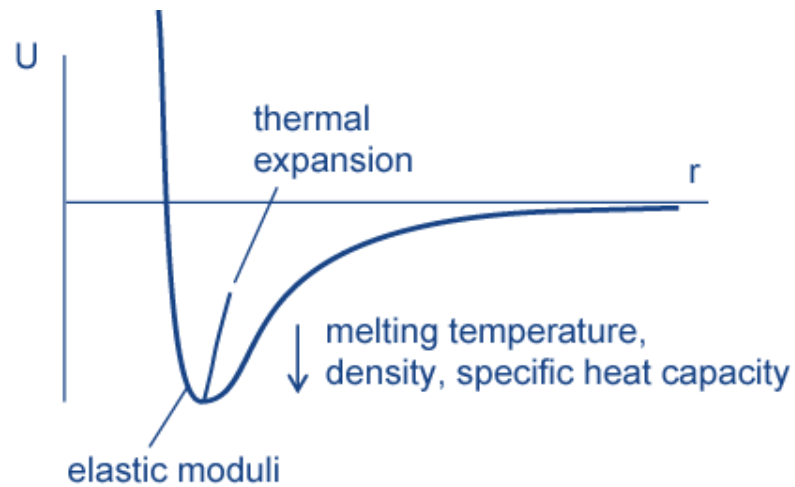
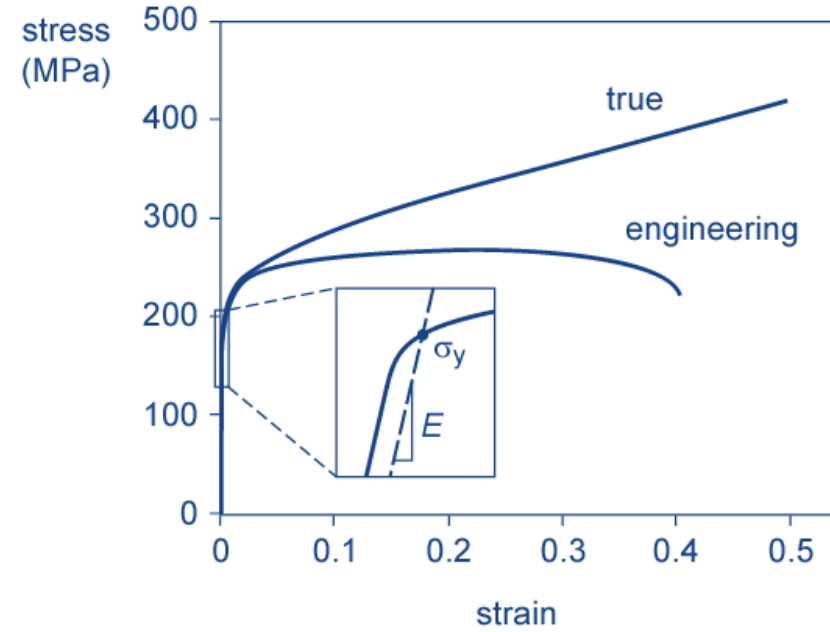
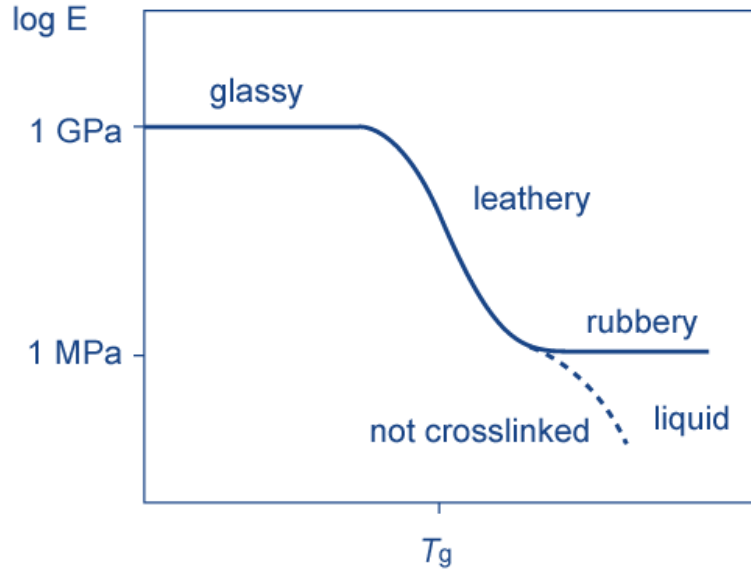
Deformation and fracture in polymers

#### 14. Order-of-magnitude values.

	<u>Metals</u>	<u>Ceramics</u>	<u>Polymers</u>
$E$ (GPa)	100	100s	0.001–1
$\nu$	0.33	0.25	up to 0.5
$K_{IC}$ (MPa m <sup>1/2</sup> )	10–100	1	1
$\gamma$ (J/m <sup>2</sup> )	1	1	<0.1
$\rho$ (g/cc)	3–20	3	1



# 15. Sketches.



Sincere thanks to:

Prof. Michael Demkowicz

Prof. Tom Eagar

Prof. Lorna Gibson

Prof. Ken Russell

Prof. Chris Schuh

Prof. Subra Suresh

Prof. Carl Thompson

Prof. Krystyn Van Vliet